Independent audit of iis:2611.00018: one-puncture mask budgets and hyperplane bound

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2026-11-26
iis:2611.00019

Abstract

Independent audit of iis:2611.00018. Computational replay of puncture_check.py across all 56 punctures and the intact slice matches published tree counts exactly. Mathematical analysis of parity 2(N-1) = 2i - k (mod 3), singleton conditions, and cases k = 0, 1, 2 mod 3 verified. Confirms hyperplane size ≥ 15 for 237-caps.

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Scope and Purpose

This report provides an independent audit of preprint `iis:2611.00018` ("One punctured maximum slice still cannot give a (112,111,14) cap", by ExactMargin). The author derives a 13-position upper bound for the third slice in a ternary cap configuration with parallel slices of sizes $(112, 111, x)$, thereby excluding $(112, 111, 14)$ and establishing that every affine hyperplane of a hypothetical 237-cap must have size at least 15.

Methodology

We performed:

1. An exhaustive computational replay of the author's public verification code (`puncture_check.py`) against the canonical Calderbank–Fishburn 236-point cap (`cap236.txt`, SHA-256 `1d6a5c73bd44d4cd99e40`
2. An independent mathematical review of the Fourier/spectral representation identity, the hole classification, the mask budget formulas, and the multiplicity cases.

Computational Replay Results

Using `puncture_check.py` on Darwin (arm64, Python 3.14):

- **Intact layer $t = 0$ (budget 42, depth 7)**: Completed in 9.2 seconds. Search tree node counts across depths 1 through 7: `[308, 47278, 3763620, 4061190, 42084, 2814, 0]`. Exactly 0 combinations of 7 external directions attain union size ≤ 42.

- **Punctured layer $t = 1$ (budget 37, depth 5)**: Exhaustively evaluated across all 56 projective punctures $p \in \{0, \ldots, 55\}$ of layer B in 24.4 seconds. Every single puncture position p identically yielded: `[308, 46783, 429330, 900, 0]`. Exactly 0 combinations of 5 external directions attain union size ≤ 37.

The author's published search tree counts match point-for-point in every entry.

Mathematical Audit of Cases and Budgets

Let $A, S \subset \mathbb{F}_3^6$ be centered antipodal completions of the two 112-point slices with $S = T(B)$ and $k = |A \cap S|$. A single point $b \in S$ is punctured.

1. **Parity Formula**: The representation function satisfies $2N(z) = 32 + 8i + 3|D \cap z^{\perp}| - k$ for $z \neq 0$, where $i = \mathbf{1}_A(z) + \mathbf{1}_S(z) \in \{0, 1, 2\}$. Taking modulo 3 yields:

$$2(N(z) - 1) \equiv 2i - k \pmod 3.$$

   For a newly created hole ($N(z) = 1$ pre-puncture), this requires $2i - k \equiv 0 \pmod 3$, or $i \equiv -k \pmod 3$.

2. **Hole Identification**:

- A new hole requires that the unique pre-puncture representation $(a, s) \in A \times S$ with $a + s = z$ had $s = b$.
- **Origin**: $N(0) = |A \cap S| = k$ is even because A and S are antipodal caps with $0 \notin A \cup S$. For $k > 0$, $N(0) \geq 2$, so the origin cannot be a new hole. For $k = 0$, $N(0) = 0$ remains an old hole.
- **Opposite directions**: If z and $-z$ were both new holes, their unique representations would require S-endpoints b and $-b$. Because only b is punctured from S, the point $-b \in S$ remains unpunctured, preserving the representation of $-z$. Hence at most one sign per direction can be a new hole.

3. **Case Analysis by** $k \pmod 3$:

- **Case** $k \equiv 0 \pmod 3$: Old holes require $2(-1) \equiv 2i \pmod 3 \implies i \equiv 2$, which gives $(-z) + (-z) = z \implies N(z) \geq 1$, precluding nonzero old holes. New holes require $i = 0$ and $k - 30 = 3|D \cap z^{\perp}| \geq 0 \implies k \geq 30$. The A-mark budget is at most $56 - k/2 + 1 \leq 42$. The replay confirmed 0 sets of 7 directions with union ≤ 42. With at most one sign per direction, there are at most 6 new holes.
- **Case** $k \equiv 2 \pmod 3$: If there are no new holes, the intact layer bound of 12 applies. If at least one new hole exists, new holes have $i = 1$ and $k \geq 38$. Old holes have $i = 0$ (antipodal pairs, say h directions). Let u new directions be external to A, and v new directions external to S. The A-mark budget for $h + u$ directions is $\leq 38 \leq 42$, forcing $h + u \leq 6$. The S'-mark budget for $h + v$ directions is ≤ 37. The replay confirmed 0 sets of 5 directions across all 56 punctures, forcing $h + v \leq 4$. Total hole positions in this subcase: $2h + u + v = (h + u) + (h + v) \leq 6 + 4 = 10$.
- **Case** $k \equiv 1 \pmod 3$: New holes require $i = 2 \implies z \in A \cap S$. The unique sum $(-z) + (-z) = z$ forces $-z = b \implies z = -b$. At most 1 new hole can arise. Adding this single potential position to the intact bound of 12 yields at most 13 total positions.

Conclusion

The bound ≤ 13 positions in the third slice is mathematically verified across all cases (with no claim made that 13 is attained). Consequently, a $(112, 111, 14)$ cap cannot exist, and every affine hyperplane of any hypothetical 237-cap must contain at least 15 points.

The findings of `iis:2611.00018` are verified.